

SU(3) dibaryons in the Einstein-Skyrme model

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(Dated: April 30, 2008)

SU(3) collective coordinate quantization to the regular solution of the $B = 2$ axially symmetric Einstein-Skyrme system is performed. For the symmetry breaking term, a perturbative treatment as well as the exact diagonalization method called Yabu-Ando approach are used. The effect of the gravity on the mass spectra of the SU(3) dibaryons and the symmetry breaking term is studied in detail. In the strong gravity limit, the symmetry breaking term significantly reduces and exact SU(3) flavor symmetry is recovered.

PACS numbers: 12.39.Dc, 21.10.-k, 04.20.-q

I. INTRODUCTION

The Skyrme model [1] is considered as an unified theory of hadrons by incorporating baryons as topological solitons of pion fields, called skyrmions. The topological charge is identified as the baryon number B . Performing collective quantization for a $B = 1$ skyrmion, one can obtain the proton and the neutron within 30% error [2]. Correspondingly multi-skyrmion ($B > 1$) solutions are expected to represent nuclei [3, 4].

The Einstein-Skyrme (ES) model can be thought of as a model of hadrons in which baryons interact with black holes. The early studies [5, 6, 7] have shown that the Schwarzschild black hole can support spherically symmetric (hedgehog) Skyrme hair which is a first counter example to the no-hair conjecture. Axially symmetric regular and black hole skyrmion solutions with $B = 2$ were found in Ref.[8]. Also some multi-skyrmion solutions $B > 2$ have been obtained [9].

If a skyrmion is regarded as a nucleon or a nuclei, it must be quantized to assign quantum numbers like spin, isospin, *etc.*. Study of gravitational effects on the quantum spectra of skyrmions was initiated in Ref. [10] by performing collective quantization of a $B = 1$ gravitating skyrmion. The study about a $B = 2$ axially symmetric skyrmion immediately followed to it [11]. It was shown there that the qualitative change in the mass difference, mean charge radius and densities under the strong gravitational influence confirms the attractive feature of gravity while the reduction of the axial coupling and transition moments indicates the gravitational effects as a stabilizer of baryons.

In Ref. [11], we investigated the gravity effects to the SU(2) solutions of the ES model and found out that the effects are seen especially in the heavier dibaryon spectra. In this paper we shall study the gravity effects to the SU(3) dibaryons because they have much larger mass compared with that of SU(2) dibaryons in terms of the

large flavor symmetry breaking. The effect will be more apparent on such dihyperons.

The extension of the Skyrme model into SU(3) have extensively been studied. Roughly speaking, solutions of the SU(3) are classified into two categories: one is embedded, another is non-embedded ones. For the non-embedded solutions, in Ref.[12] the classical SO(3) chiral field is simply extended into three flavor space. The authors constructed the solutions in the ES system and found particle-like and black hole solutions. More systematic analysis in this direction has been done in Refs.[9, 13]. By using the harmonic map ansatz, the authors have investigated various large flavor and multi winding number solitons.

On the other hand, for the embedded solutions, the successful approaches are based on perturbations. In the bound state approach [14] static baryons are considered as bound states of the kaon and the skyrmions. The chiral field is expanded in the kaon fluctuations around the classical solutions. This approximation works well for the large symmetry breaking. Collective coordinate approach is essentially a natural extension of the SU(2) collective quantization scheme, including symmetry breaking terms. Good quantum numbers for spin and isospin are obtained by quantizing the their rotational zero modes [15, 16]. Yabu-Ando (YA) treatment [17] is a sort of the collective quantization but the collective Hamiltonian is exactly diagonalized by using Euler-angle parameterization of the SU(3) rotations. As a result, baryon appears to be its lowest irreducible representation (irrep) but contain significant admixture of higher representations. The application to the $B = 2$ axially symmetric skyrmions [18] and to the multi-skyrmions ($3 \leq B \leq 8$) [19] in flat space have been studied. They observed large mixing of higher irreps especially in excited or higher winding number states.

In this Letter, we construct SU(3) axially symmetric skyrmions in the ES system in terms of the collective quantization and YA approach. We estimate mass spectra of the dibaryons down to strangeness $S = -6$. Since our concern is about contributions of higher irreps to the lowest states and we shall show change of the mixing probabilities about varying coupling strength of the

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gravity.

II. CLASSICAL GRAVITATING $B = 2$ SKYRMIONS

In this section we shall introduce basic formalism and classical solutions of the axially symmetric gravitating skyrmions. The SU(3) extended Skyrme Lagrangian coupled with gravity can be written as

$$L = L_G + L_S + L_{SB} + L_{WZ} \quad (1)$$

where L_G is the standard Einstein-Hilbert Lagrangian

$$L_G = \int d^3r \sqrt{-g} \frac{1}{16\pi G} R \quad (2)$$

and the remaining parts are about the SU(3) extension of the Skyrme Lagrangian, which are defined in terms of the chiral field $U(x) \in \text{SU}(3)$ and using the notation $l_\mu = U^\dagger \partial_\mu U$ as follows:

$$L_S = \int d^3r \sqrt{-g} \left[\frac{1}{16} F_\pi^2 g^{\mu\nu} \text{Tr}(l_\mu l_\nu) + \frac{1}{32e^2} g^{\mu\rho} g^{\nu\sigma} \text{Tr}([l_\mu, l_\nu][l_\rho, l_\sigma]) \right], \quad (3)$$

$$L_{SB} = \frac{1}{16} F_\pi^2 m_\pi^2 \int d^3r \sqrt{-g} \text{Tr}(U + U^\dagger - 2) + \frac{1}{24} (F_\kappa^2 m_\kappa^2 - F_\pi^2 m_\pi^2) \int d^3r \sqrt{-g} \text{Tr}(1 - \sqrt{3}\lambda_8) \times (U + U^\dagger - 2), \quad (4)$$

$$L_{WZ} = -\frac{iN_c}{240\pi^2} \int_Q d\Sigma^{\mu\nu\lambda\rho\sigma} \text{Tr}[l_\mu l_\nu l_\lambda l_\rho l_\sigma], \quad (5)$$

where F_π and e are basic model parameters which indicate the pion decay constant, a dimensionless parameter, respectively. The L_{SB} is comprised of all chiral and flavor symmetry breaking terms. The L_{WZ} is usual Wess-Zumino term which concerns with the topological charge of the soliton. F_κ, m_κ are the kaon decay constant and the mass. N_c means a number of color.

In this Letter, we employ the solution of the chiral field with axially symmetric, winding number two which is considered as a possible candidate for the $B = 2$ minimal energy configuration [3]

$$U_0(\mathbf{r}) = \exp[iF(r, \theta)\boldsymbol{\tau} \cdot \mathbf{n}_R]. \quad (6)$$

where \mathbf{n}_R is defined as

$$\mathbf{n}_R = (\sin \Theta(r, \theta) \cos n\varphi, \sin \Theta(r, \theta) \sin n\varphi, \cos \Theta(r, \theta)) \quad (7)$$

and $n \in \mathbb{Z}$ is the winding number. We explore the solution with $n = 2$. SU(3) chiral field is constructed by trivial embedding:

$$U(\mathbf{r}) = \begin{pmatrix} U_0(\mathbf{r}) & 0 \\ 0 & 1 \end{pmatrix}. \quad (8)$$

Correspondingly, the following axially symmetric ansatz is imposed on the metric [20]

$$ds^2 = -f dt^2 + \frac{m}{f} (dr^2 + r^2 d\theta^2) + \frac{l}{f} r^2 \sin^2 \theta d\varphi^2 \quad (9)$$

where the metric functions f , m and l are the function of coordinates r and θ . This metric is symmetric with respect to the z -axis ($\theta = 0$). Substituting these ansatz to the Lagrangian (1), one obtains the following static (classical) energy for the chiral fields

$$M_{class} = 2\pi \frac{F_\pi}{e} \int dx d\theta \left[\frac{\sqrt{l} \sin \theta}{8} \times \{ x^2 ((\partial_x F)^2 + (\partial_x \Theta)^2 \sin^2 F) + (\partial_\theta F)^2 + (\partial_\theta \Theta)^2 \sin^2 F + \frac{n^2 m}{l \sin \theta} \sin^2 F \sin^2 \Theta \} + \frac{\sqrt{l} \sin \theta}{2} \left[\frac{f}{m} (\partial_x F \partial_\theta \Theta - \partial_\theta F \partial_x \Theta)^2 \sin^2 F + \frac{n^2 f}{l \sin^2 \theta} \sin^2 F \sin^2 \Theta \{ ((\partial_x F)^2 + \frac{1}{x^2} (\partial_\theta F)^2) + ((\partial_x \Theta)^2 + \frac{1}{x^2} (\partial_\theta \Theta)^2) \sin^2 F \} \right] + \frac{1}{4} \frac{m\sqrt{l}}{f} x^2 \sin \theta \beta_\pi^2 (1 - \cos F) \right], \quad (10)$$

where dimensionless variable $x = eF_\pi r$ and $\beta_\pi = \frac{m_\pi}{eF_\pi}$ are introduced.

For the profile functions, the boundary conditions at the $x = 0, \infty$ are imposed

$$F(0, \theta) = \pi, \quad F(\infty, \theta) = 0, \quad (11)$$

$$\partial_x \Theta(0, \theta) = \partial_x \Theta(\infty, \theta) = 0. \quad (12)$$

At $\theta = 0$ and $\pi/2$,

$$\partial_\theta F(x, 0) = \partial_\theta F(x, \frac{\pi}{2}) = 0, \quad (13)$$

$$\Theta(x, 0) = 0, \quad \Theta(x, \frac{\pi}{2}) = \frac{\pi}{2}. \quad (14)$$

For the solutions to be regular at the origin $x = 0$ and to be asymptotically flat at infinity, the following boundary conditions must be imposed

$$\partial_x f(0, \theta) = \partial_x m(0, \theta) = \partial_x l(0, \theta) = 0, \quad (15)$$

$$f(\infty, \theta) = m(\infty, \theta) = l(\infty, \theta) = 1. \quad (16)$$

For the configuration to be axially symmetric, the following boundary conditions must be imposed at $\theta = 0$ and $\pi/2$

$$\partial_\theta f(x, 0) = \partial_\theta m(x, 0) = \partial_\theta l(x, 0) = 0, \quad (17)$$

$$\partial_\theta f(x, \frac{\pi}{2}) = \partial_\theta m(x, \frac{\pi}{2}) = \partial_\theta l(x, \frac{\pi}{2}) = 0. \quad (18)$$

The covariant topological current is defined by

$$B^\mu = \frac{\epsilon^{\mu\nu\rho\sigma}}{24\pi^2} \frac{1}{\sqrt{-g}} \text{tr}(U^{-1} \nabla_\nu U U^{-1} \nabla_\rho U U^{-1} \nabla_\sigma U). \quad (19)$$

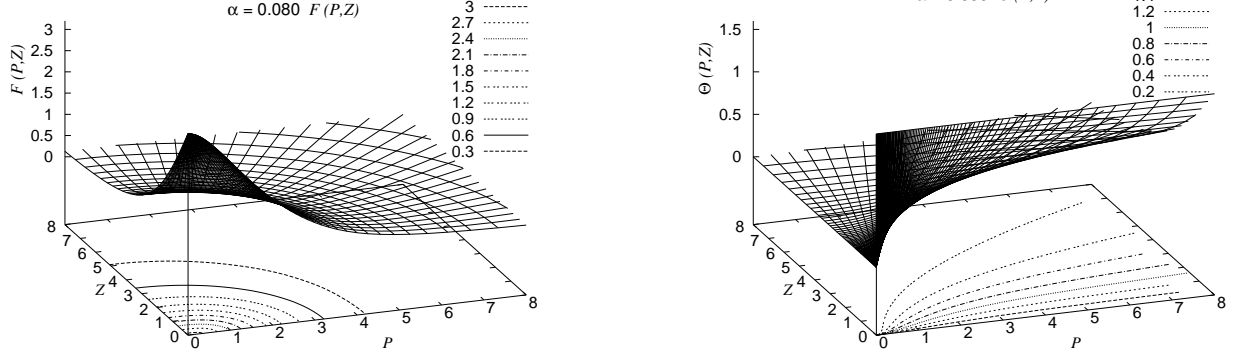


FIG. 1: The profile functions F, Θ for $\alpha = 0.080$ in the cylindrical coordinate with dimensionless variables $P := ef_\pi \rho, Z = ef_\pi z$.

Substituting the ansatz (6),(7) into (19) the zeroth component is estimated as

$$B^0 = -\frac{1}{\pi^2 \sqrt{-g}} \sin^2 F \sin \Theta (\partial_x F \partial_\theta \Theta - \partial_\theta F \partial_x \Theta). \quad (20)$$

The baryon number of the soliton B is derived from its spatial integral, thus

$$B = \int d^3r \sqrt{-g} B^0 = \frac{1}{2\pi} (2F - \sin 2F) \cos \Theta \Big|_{F_0, \Theta_0}^{F_1, \Theta_1}. \quad (21)$$

The inner and outer boundary conditions $(F_0, \Theta_0) = (\pi, 0)$ and $(F_1, \Theta_1) = (0, \pi)$ yield $B = 2$.

By taking a variation of the static energy (10) with respect to F and Θ , one obtains the equations of motion for the profile functions. The field equations for the metric functions f, m and l are derived from the Einstein equations. The explicit form of the equations is essentially same (except for contribution of the mass term) as reported in Ref.[11].

The effective coupling constant of the Einstein-Skyrme system is given by

$$\alpha = 4\pi G F_\pi^2 \quad (22)$$

which is the only free parameter.

To solve the equations of motion numerically, the relaxation method with the typical grid size 100×30 are performed. We observe that the solution survives at $0 \leq \alpha \leq 0.120$. Including the mass term, the range becomes a little narrow. We show examples of our numerical results for the profile functions F, Θ in Fig.1 and also for the metric functions f, l, m in Fig.2.

III. THE SU(3) COLLECTIVE QUANTIZATION

We study the SU(3) extension of the axially symmetric $B = 2$ skyrmions by Yabu-Ando approach together with naive collective coordinate quantization. SU(3) chiral field is constructed by trivial embedding

$$\tilde{U}(\mathbf{r}, t) = A(t) \begin{pmatrix} U_0(R(t)\mathbf{r}) & 0 \\ 0 & 1 \end{pmatrix} A^\dagger(t) \quad (23)$$

where U_0 is introduced in Eq.(6). $A(t)$ is time dependent SU(3) rotational matrix and $R(t)$ describes a spatial rotation of the soliton. We introduce the angular velocities Ω_a, ω_l which are defined by

$$A^\dagger \dot{A} = \frac{i}{2} \sum_{a=1}^8 \lambda_a \Omega_a, \quad (24)$$

$$(\dot{R}R^\dagger)_{ik} = \sum_{l=1}^3 \varepsilon_{ikl} \omega_l. \quad (25)$$

Substituting the chiral field (23) into the Lagrangians (3)-(5) and after a lengthy calculation, one finally obtain the effective Lagrangian of the form:

$$L = -M_{class} + \frac{1}{2} I_N \sum_{p=1}^2 \Omega_p^2 + \frac{1}{2} I_J \sum_{p=1}^2 \omega_p^2 + \frac{1}{2} I_3 (\Omega_3^2 + n\omega_3^2) + \frac{1}{2} I_S \sum_{k=4}^7 \Omega_k^2 - \frac{N_c}{2\sqrt{3}} \Omega_8 + \frac{1}{2} \gamma (1 - D_{88}(A)), \quad (26)$$

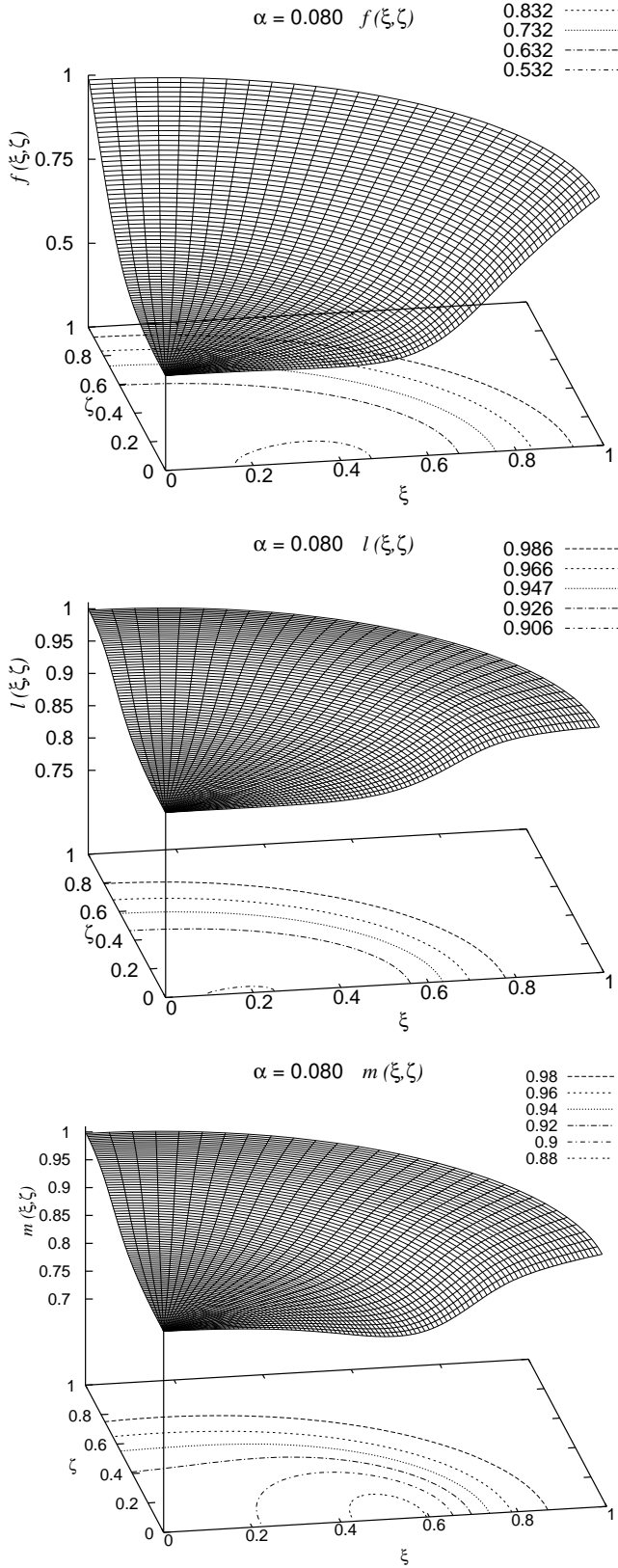


FIG. 2: The metric functions f, l, m in the cylindrical coordinate with dimensionless, rescaled variables $\xi = P/(1+P), \zeta = Z/(1+Z), P = ef_\pi\rho, Z = ef_\pi z$.

where I_N, I_J, I_3, I_S are called the moments of inertia and their explicit forms are

$$I_N = \frac{\pi}{F_\pi e^3} \int dx d\theta \left[\frac{m\sqrt{l}}{4f^2} x^2 \sin\theta \sin^2 F (1 + \cos^2 \Theta) + \frac{\sqrt{l}}{f} \sin\theta \sin^2 F \{ (1 + \cos^2 \Theta) (x^2 (\partial_x F)^2 + (\partial_\theta F)^2) + \sin^2 F \cos^2 \Theta (x^2 (\partial_x \Theta)^2 + (\partial_\theta \Theta)^2) + \frac{n^2 m}{l \sin^2 \theta} \sin^2 F \sin^2 \Theta \} \right], \quad (27)$$

$$I_3 = \frac{\pi}{F_\pi e^3} \int dx d\theta \left[\frac{m\sqrt{l}}{2f^2} x^2 \sin\theta \sin^2 F \sin^2 \Theta + \frac{2\sqrt{l}}{f} \sin\theta \sin^2 F \sin^2 \Theta \{ x^2 (\partial_x F)^2 + (\partial_\theta F)^2 + \sin^2 F (x^2 (\partial_x \Theta)^2 + (\partial_\theta \Theta)^2) \} \right], \quad (28)$$

$$I_J = \frac{\pi}{F_\pi e^3} \int dx d\theta \left[\frac{m\sqrt{l}}{4f^2} x^2 \sin\theta ((\partial_x F)^2 + (\partial_\theta \Theta)^2 \sin^2 F + n^2 \cot^2 \theta \sin^2 F \sin^2 \Theta) + \frac{\sqrt{l}}{f} x^2 \sin\theta \sin^2 F \{ (\partial_x F \partial_\theta \Theta - \partial_\theta F \partial_x \Theta)^2 + n^2 ((\partial_x F)^2 + (\partial_x \Theta)^2 \sin^2 F) \cot^2 \theta \sin^2 \Theta \} + \frac{n^2 \sqrt{l}}{f \sin \theta} (\cos^2 \theta + \frac{m}{l}) \times ((\partial_\theta F)^2 + (\partial_\theta \Theta)^2 \sin^2 F) \sin^2 F \sin^2 \Theta \right], \quad (29)$$

$$I_S = \frac{\pi}{F_\pi e^3} \int dx d\theta (1 - \cos F) \left[\frac{m\sqrt{l}}{4f^2} x^2 \sin\theta + \frac{\sqrt{l}}{4f} x^2 \sin\theta \{ (\partial_x F)^2 + \sin^2 F (\partial_x \Theta)^2 \} + \frac{\sqrt{l}}{4f} \sin\theta \{ (\partial_\theta F)^2 + \sin^2 F (\partial_\theta \Theta)^2 \} + \frac{mn^2}{l \sin^2 \theta} \sin^2 F \sin^2 \Theta \right]. \quad (30)$$

$\frac{1}{2}\gamma(1 - D_{88})$ exhibits strength of the symmetry breaking and the explicit form of γ is

$$\gamma = \frac{2\pi F_\pi}{3e} (\beta_\kappa^2 - \beta_\pi^2) \int dx d\theta x^2 \sin\theta (\cos F - 1), \quad (31)$$

where $\beta_\kappa = \frac{m_\kappa F_\pi}{e F_\pi^2}$. D_{88} is a component of Wigner function which is defined as

$$D_{ab}(A) = \frac{1}{2} \text{Tr}(\lambda_a A^\dagger \lambda_b A). \quad (32)$$

From (26) the Hamiltonian reads

$$H = M_{class} + \frac{J(J+1)}{2I_J} + \frac{1}{2} \left(\frac{1}{I_N} - \frac{1}{I_S} \right) N(N+1) + \frac{1}{2} \left(\frac{1}{I_3} - \frac{1}{I_N} - \frac{n^2}{I_J} \right) L^2 - \frac{3}{8I_S} B^2 + \frac{1}{2I_S} \varepsilon_{SB} \quad (33)$$

where eigenvalues of diagonal operators are already inserted. Here, the eigenvalue of J is spin, I is isospin, N is right isospin derived from $N = \frac{1}{2}p_0$ where (p_0, q_0) is the minimal irrep and L is the third component of the body fixed spin operator which determine parity P of the

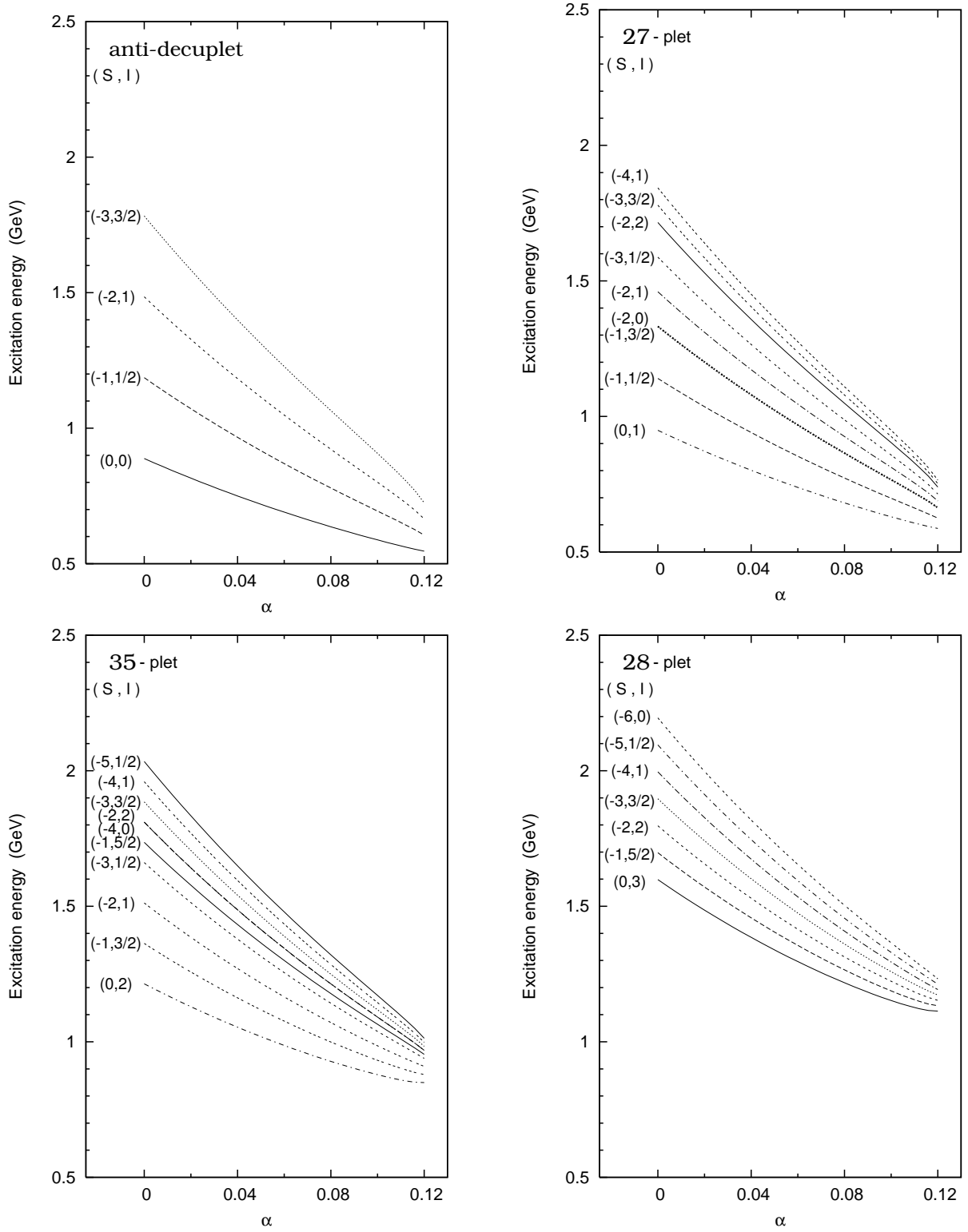


FIG. 3: The coupling constant dependence of the mass difference from the classical energy for the multiplets $\{\overline{10}\}, \{27\}, \{35\}, \{28\}$ $((p, q) = (0, 3), (2, 2), (4, 1), (6, 0), \text{respectively})$ are shown in the unit of GeV. The quantized energy is computed in terms of the naive perturbation technique.

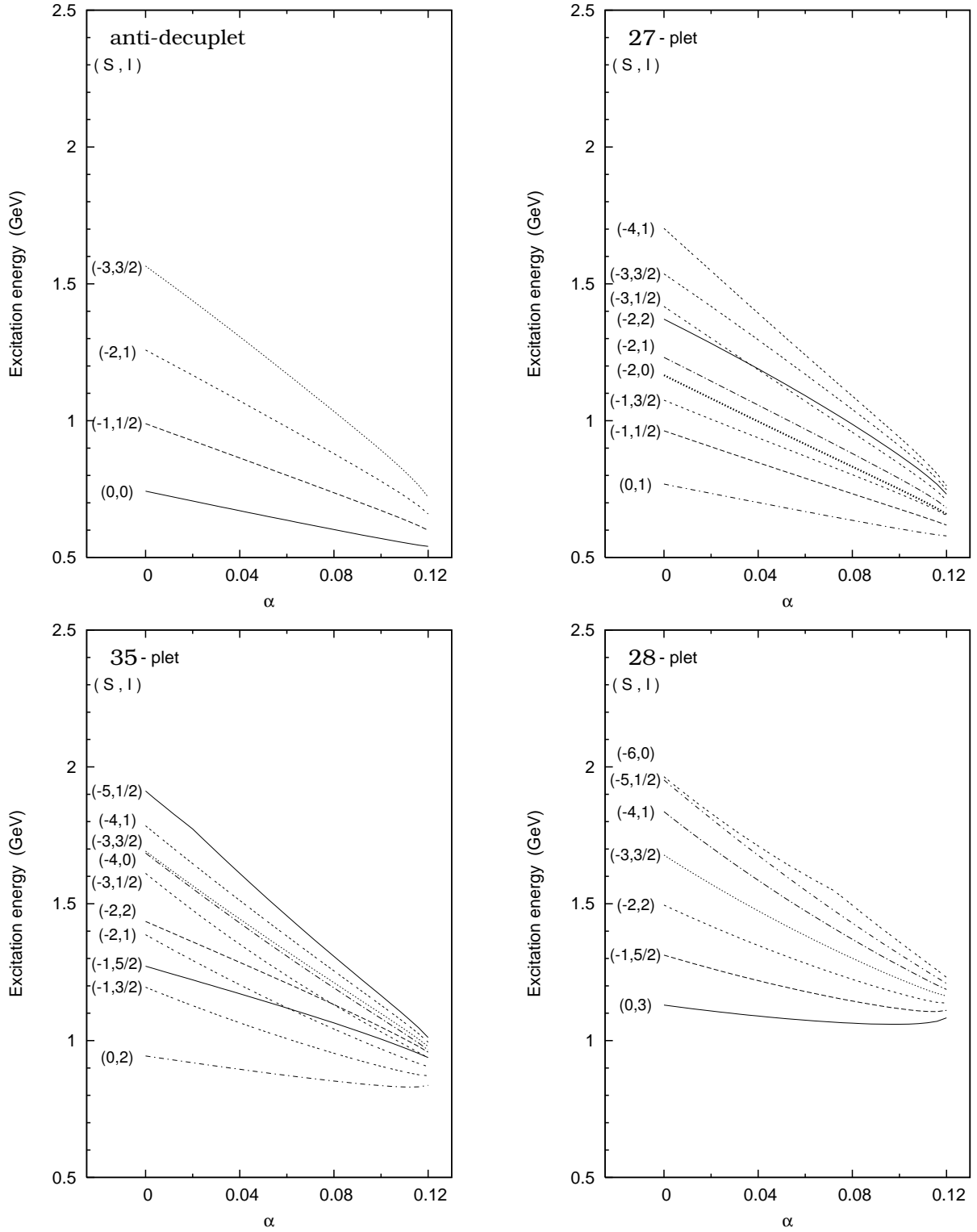


FIG. 4: The coupling constant dependence of the mass difference from the classical energy for the multiplets $\{\overline{10}\}, \{27\}, \{35\}, \{28\}$ ($(p, q) = (0, 3), (2, 2), (4, 1), (6, 0)$, respectively) are shown in the unit of GeV. The quantized energy is computed in terms of YA.

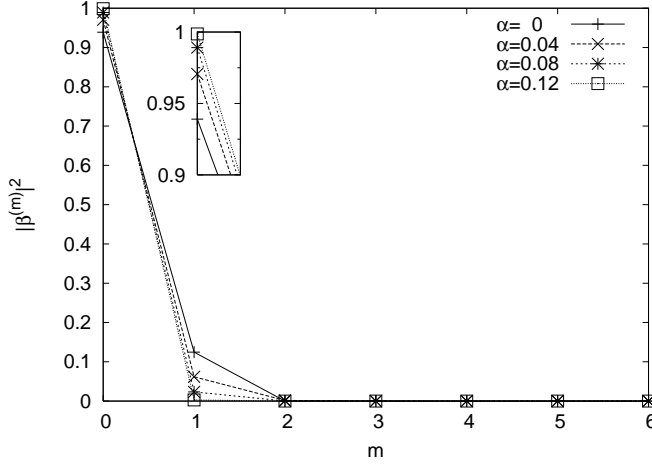


FIG. 5: Mixing probability of the higher representations to the lowest energy state of NN channel in $\{\overline{10}\}$.

state by the relation of $P = (-1)^L$. The eigenvalue of the ε_{SB} is derived from following eigenequation

$$\{C_2[SU(3)] + I_S \gamma (1 - D_{88})\} \Psi = \varepsilon_{SB} \Psi \quad (34)$$

where $C_2[SU(3)]$ is Casimir operator of $SU(3)$.

We shall investigate (34) in two folds. One is to treat the symmetry breaking term perturbatively, another is to diagonalize the whole via a basis of the $SU(3)$ Wigner functions.

A. Perturbative method

If symmetry breaking effect γ is certainly small, perturbative treatment seems to be good approximation. We introduce a wave function of the form [21]

$$\begin{aligned} \Psi &:= \Phi_{II_3 Y, NN_3 Y_R, J J_3}^{(m)}(A) D_{J_3, -n N_3}^{J*}(R^{-1}) \quad (35) \\ \Phi_{II_3 Y, NN_3 Y_R, J J_3}^{(m)}(A) &= \sqrt{d^{(m)}} (-1)^{\frac{Y_R}{2} + N_3} \\ &\times D_{II_3 Y, NN_3 Y_R}^{(m)*}(A^{-1}) \quad (36) \end{aligned}$$

where the dimension of the (p, q) irrep is expressed by $d^{(m)} = (p+1)(q+1)(p+q+2)/2$, the m is representation of $SU(3)$ group, and subscript of the Y and Y_R is hypercharge and right hypercharge respectively.

With the operation of the collective quantization, the angular velocity Ω_8 appears linear in Eq.(26). Therefore we obtain a constraint

$$Y_R = \frac{1}{3} N_c B, \quad (37)$$

which means that the symmetry $U_R(1)$ is redundant. Thus we obtain $Y_R = 2$.

In terms of Eq.(36), the expectation value of the Casimir invariants $C_2(SU(3))$ in Eq.(34) is easily obtained

$$\langle C_2(SU(3)) \rangle = \frac{1}{3} (p^2 + q^2 + pq + 3(p+q)). \quad (38)$$

For the symmetry breaking term $I_S \gamma (1 - D_{88})$, the estimation of the expectation value can be done by performing the integral of the three Wigner rotation matrices [22, 23] which is evaluated by the $SU(3)$ Clebsch-Gordan coefficient, or the isoscalar factor

$$\begin{aligned} &\int dA D_{\nu_3 \nu'_3}^{(m_3)*}(A) D_{\nu_1 \nu'_1}^{(m_1)}(A) D_{\nu_2 \nu'_2}^{(m_2)}(A) \\ &= \frac{1}{d^{(m)}} \sum_{\mu} \begin{pmatrix} m_1 & m_2 & m_{3\mu} \\ \nu_1 & \nu_2 & \nu_3 \end{pmatrix} \begin{pmatrix} m_1 & m_2 & m_{3\mu} \\ \nu'_1 & \nu'_2 & \nu'_3 \end{pmatrix} \quad (39) \end{aligned}$$

Computations of the Clebsch-Gordan coefficients can be performed by the numerical algorithm of Ref. [24].

B. Diagonalization of the collective Hamiltonian

If symmetry breaking effect is crucial, the naive perturbation will substantially fail. Yabu-Ando approach can improve the situation. In YA, state of baryon appears to be its lowest irrep but contain large admixture of higher irreps. We shall see that such mixing reduces in large gravity limit.

The wave function of the Hamiltonian is expanded in terms of a wave function of the lowest representation (36)

$$\Psi := \sum_m \beta^{(m)} \Phi_{II_3 Y, NN_3 Y_R, J J_3}^{(m)}(A) D_{J_3, -n N_3}^{J*}(R^{-1}) \quad (40)$$

In terms of the basis, the eigenvalue problem in Eq.(34) can be reduced to a matrix diagonalization problem.

IV. NUMERICAL RESULTS

For the actual calculations, we fix $F_\pi = 108$ MeV, $e = 4.84$, $\beta_\pi = 0.263$. The kaon decay constant, experimentally, is $F_\kappa \approx \sqrt{2} F_\pi$, but for the simplicity, we employ $F_\kappa = F_\pi$. For the kaon mass, we employ the experimental value, *i.e.*, $\beta_\kappa = 0.952$.

We estimate mass spectra belonging to $SU(3)$ multiplets $\{\overline{10}\}, \{27\}, \{35\}, \{28\}$ (or in the (p, q) representation, $(0, 3), (2, 2), (4, 1), (6, 0)$, respectively). The Finkelstein-Rubinstein constraints [25] tells us that for $\{\overline{10}\}, \{35\}$, $J = 1$ is chosen for the ground state, otherwise one can set $J = 0$ [26]. The eigenvalue of L concerns with the third component of body fixed spin operator [3]. Substantially it is related to the orbital angular momentum but no experimental identification has been done. Therefore in our analysis we put $L = 0$ for all multiplets states.

In YA treatment, one needs to truncate the base in finite size. We expand the collective wave function with $N \leq 3$, except for the states $(S, I) = (-6, 0)$ in $\{28\}$, $(-4, 0)$, $(-4, 1)$ and $(-5, 1/2)$ in $\{35\}$. In those states, we expand the base with $N \leq 4$ for obtaining sufficient convergence.

In Fig.3 presents the α dependence of the mass spectra within the naive perturbation scheme. Actually, we show mass difference between the quantized mass spectra and the classical energy. Also Fig.4 presents the results of Yabu-Ando treatment. In both results the spectra as well as their differences within each multiplet decrease monotonically with increasing α . On the other hand, mass differences between different multiplet but with same quantum numbers (S, I) increase, which have been already observed in the calculation of $SU(2)$ [11]. One easily observe that difference between the results of two treatments disappears with increasing α . This behavior is easily understood. In Fig.5 we illustrate the mixing probability of the multiplet for NN channel in $\{10\}$ for various α . For increasing α , the mixing of higher representations are significantly decreased; as a result, the naive perturbation is sufficient for the analysis of $SU(3)$ dibaryons for such strong gravity region. In our analysis, the pion and the kaon mass and the coupling constants are fixed by their experimental values (we simply set $F_\kappa = F_\pi$ for the coupling constant) and if we take into account variations of the mesonic data about change of gravity, exact $SU(3)$ flavor symmetry will attain at a strong gravity limit.

V. CONCLUSION

In this Letter, we have studied the gravitational effect to the dibaryons in the axially symmetric ES model. In particular, we have investigated gravity coupling con-

stant dependence of the energy spectra of the $SU(3)$ dibaryons. We have used the collective quantization in three flavor space. To treat the symmetry breaking term, we employ the lowest order (naive) perturbation to that as well as Yabu-Ando treatment. Both treatments have shown that mass differences between spectra with different strangeness decrease monotonically and increase within different multiplet but with same quantum numbers (S, I) with increasing α . In the strong gravity limit, the $SU(3)$ flavor symmetry recovers; all the spectra degenerate in each multiplet and no mixing between the multiplets occur. Such symmetry restorations may be observed in high energy experiment at LHC.

In this Letter, we treat α as a free parameter. In the Einstein-Skyrme theory, the Planck mass is related to the pion decay constant F_π and coupling constant α by $M_{pl} = F_\pi \sqrt{4\pi/\alpha}$. To realize the realistic value of the Planck mass, the coupling constant should be extremely small with $\alpha \sim O(10^{-39})$. However, we have shown that the effects of gravity can be observed only in large α . Some theories such as scalar-tensor gravity theory [27] and “brane world scenario”[28] predict large enhancement of the gravitational constant (in other words the reduction of 4 dim. Planck mass). There may have been an epoch in the early universe and may observe at an ultra high energy experiment where the gravitational effects on hadrons are crucial.

Acknowledgments

We would like to thank Rajat K.Bhaduri for drawing our attention to this subject and useful comments. Also we deeply appreciate to Noriko Shiiki for valuable discussions.

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